0.1 Learning Objectives

By the end of this lecture, you will be able to:

- Distinguish one-sample from two-sample t-tests
- Apply the five-step hypothesis testing procedure to a real dataset
- Understand the structure and goals of the course project

1. Worked Example: Hypothesis Testing

1.1 Quick Review: Two-Sample t-Test

Last lecture we compared **two groups**

1.1 Quick Review: Two-Sample t-Test

bikers vs. drivers:

$$t = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

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Two-sample t-test

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Today: a worked example of each.

1.2 The Scenario

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A barista is skeptical. She times $n = 12$ consecutive shots during a morning rush:

Shot	1	2	3	4	5	6
Time (s)	24.2	26.1	25.8	23.9	27.0	24.5
Shot	7	8	9	10	11	12
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Is there evidence that the true mean extraction time is greater than 25 seconds?

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$$H_0 : \mu = 25 \quad \text{seconds}$$

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This is a **one-tailed test**

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$$s^2 \approx 0.937 \quad \rightarrow \quad s \approx 0.968$$

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Degrees of freedom: $\mathrm{df} = n - 1 = 11$

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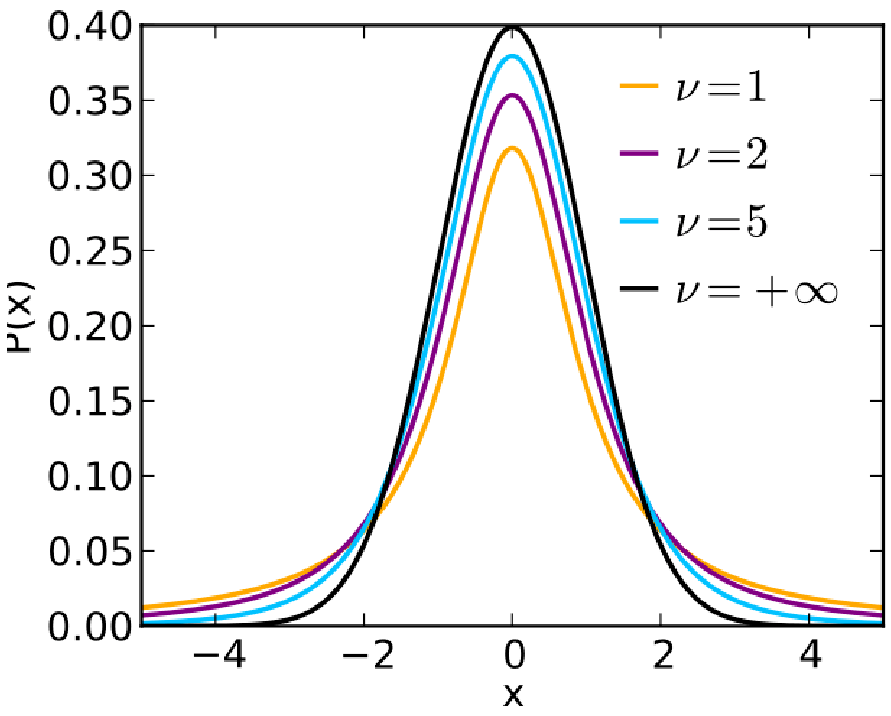
$$p \approx 0.09$$



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Note: At the 5% significance level, there is **not enough evidence** to conclude that the mean extraction time exceeds 25 seconds.

The sample mean of 25.4s is higher, but with $n = 12$ observations and this variability, the difference could plausibly be due to chance.

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Note: Same sample mean, same standard deviation — but **more data** gives us more confidence. Larger n means smaller standard error $\frac{s}{\sqrt{n}}$, which makes the t-statistic larger.

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4. **Find the p-value:** from the t-distribution with $n-1$ df
5. **Decide:** if $p < \alpha$, reject H_0; otherwise, fail to reject

1.10 Try It: Two-Sample Problem

Two sections of CHE 2A take the same final exam. Section A ($n_1 = 35$) averages 74 with $s_1 = 12$. Section B ($n_2 = 40$) averages 78 with $s_2 = 10$.

Is there a significant difference between the sections at $\alpha = 0.05$?

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Try it yourself

Talk to your neighbor and try to solve this problem.

1.11 Solution

$$t = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{74 - 78}{\sqrt{\frac{144}{35} + \frac{100}{40}}} = \frac{-4}{\sqrt{4.11 + 2.5}} = \frac{-4}{2.57} \approx -1.56$$

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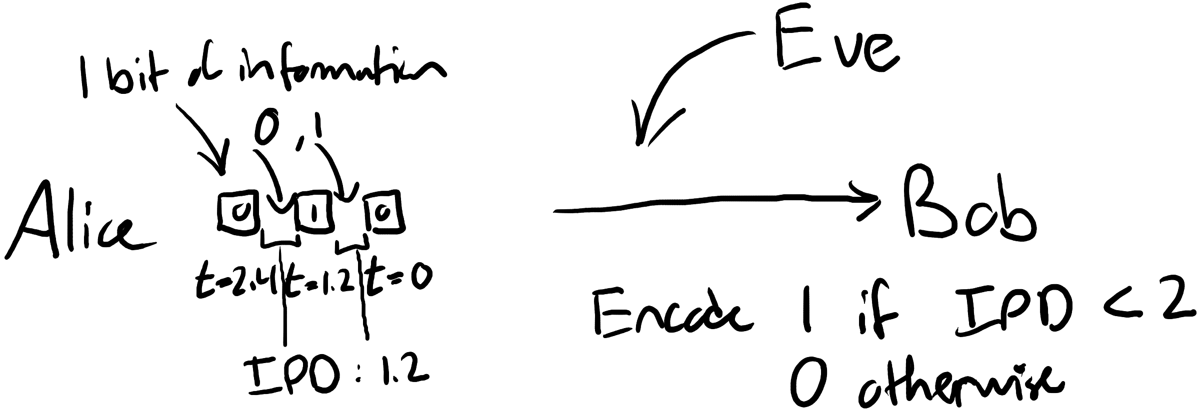
Fail to reject H_0. Not enough evidence to say the sections differ.

2. Course Project: Covert Timing Channels



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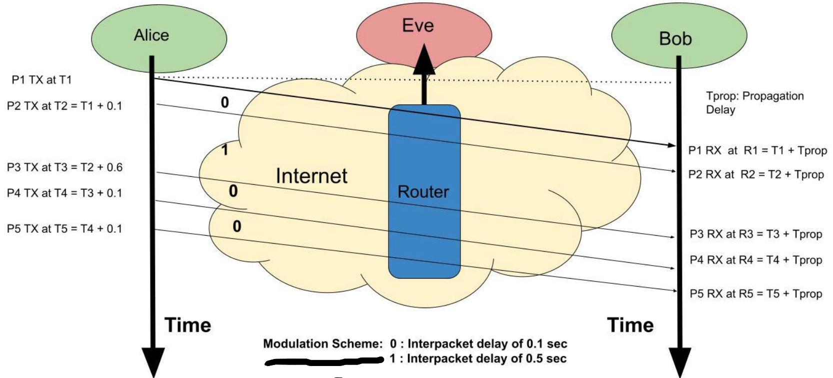
The **inter-packet delay** (IPD) is the time between consecutive packets. Alice encodes bits by choosing different delays:

Bit	IPD	Meaning
0	0.1 sec	Short delay
1	0.5 sec	Long delay



FM

AM

Bit

0 $IPD \sim N(0.1, 0.05)$

1 $IPD \sim N(0.5, 0.05)$

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QQ-plots — compare whether two datasets come from the same distribution

Simulation — model the source, buffer, and sender as a system

Distributions — exponential and uniform IPDs

2.3 Project Structure

Part 1: Design

(35 pts)

Implement encoding schemes and compare IPD distributions

Part 2: Detection

(40 pts)

Use QQ-plots to evaluate whether covert traffic is detectable

Part 3: Implementation

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Simulate the full system with a packet buffer

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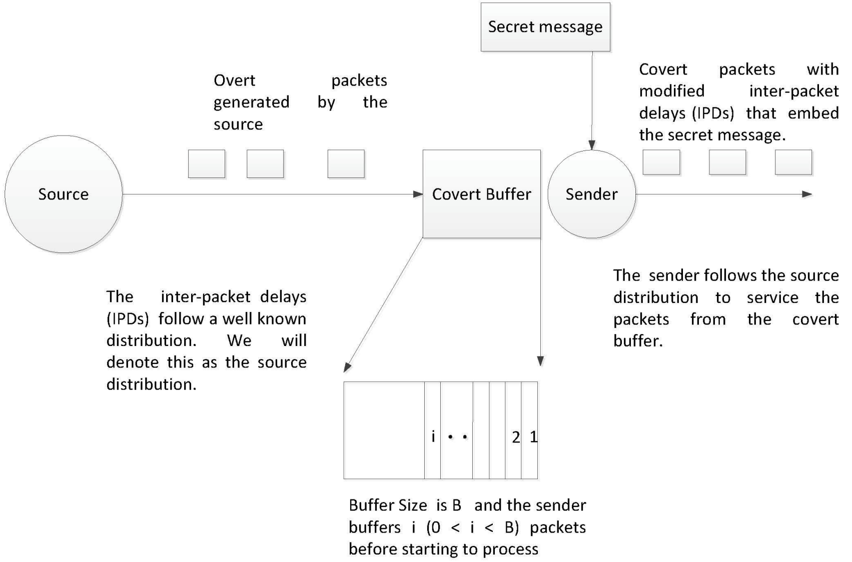

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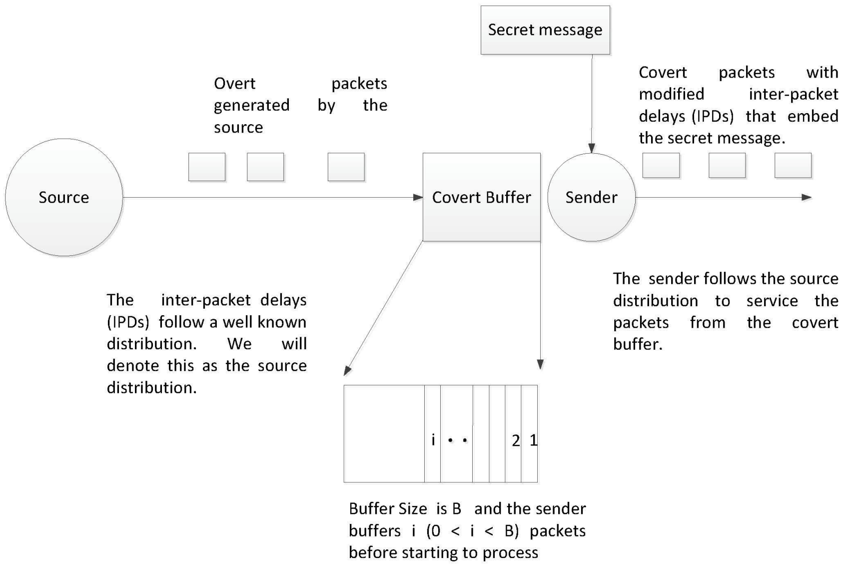

2.3 Project Structure

Start early

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Part 3 involves simulation that requires careful debugging.

2.4 Recap

Today we covered:

- **One-sample t-test:** compare a sample mean to a claimed value μ_0
- **Two-sample t-test:** compare two group means (from last lecture)
- **Five steps:** hypotheses, statistics, t-statistic, p-value, decision
- **More data** (larger n) gives more statistical power
- **Course project:** covert timing channels — design, detect, and simulate