ECS171: Machine Learning

L6
Supervised Classifiers

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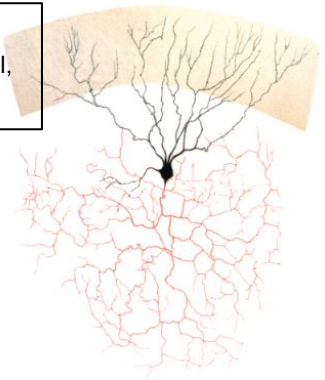
MOCO Amsterdam garden

Intended Learning Outcomes

- Discuss pros/cons of various activation functions and apply them in simple examples
- Discuss aspects of neural networks e.g. layers and nodes
- Describe and apply feature engineering methodologies such as normalisation and label encoding (one-hot and one-output)
- Describe and demonstrate the workings of a feed forward neural network in terms of how a single node is updated (linear combination of inputs and activation function)

The neuron

Golgi's drawing of a cerebellar Type II cell, now called a Golgi cell

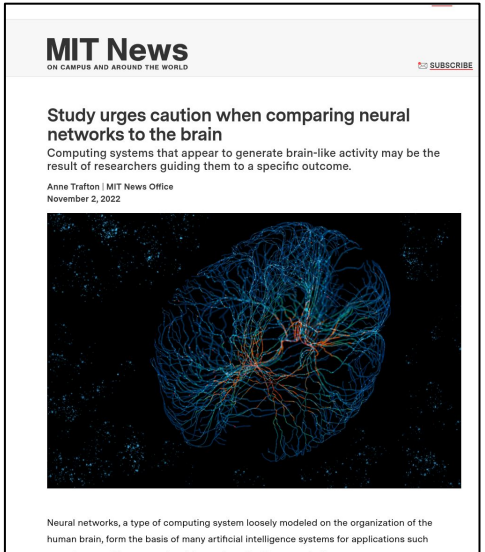


Camillo Golgi (Italy) 1843-1926

Santiago Ramon Cajal (Spain) 1852-1934

- Nobel prize 1906: Golgi (method) & Cajal (insights) for structure of the brain and spinal cord
- Functional localization: Different parts of e.g. the nervous system
- The neuron doctrine: The brain and spinal cord are made up of individual elements, called neurons.
  - Neurons touch one another, but don't fuse

Beware anthropomorphism



MIT News

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Study urges caution when comparing neural networks to the brain

Computing systems that appear to generate brain-like activity may be the result of researchers guiding them to a specific outcome.

Anne Trafton | MIT News Office
November 2, 2022

Neural networks, a type of computing system loosely modeled on the organization of the human brain, form the basis of many artificial intelligence systems for applications such

Anthropomorphism: the attribution of human characteristics or behavior to a god, animal, or object

Perceptron Learning Algorithm recapitulation

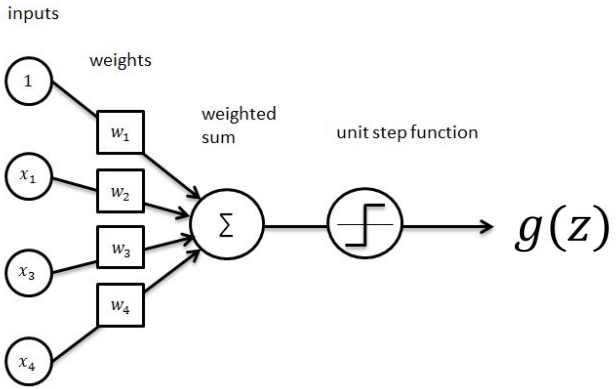
- The Perceptron Learning Algorithm is supervised learning that is good for binary classification
- The Perceptron model takes an input, aggregates it (calculates the weighted sum), and with the step function returns 1 if this is more than a threshold or 0 if it is equal or below

$$w_j = w_j + a\left(y^{(i)} - g\left(x^{(i)}; w\right)\right)x_j^{(i)}$$

$$w_j = w_j + a\left(y^{(i)} - g(z)\right)x_j^{(i)}$$

$$z = w^T x = w_0 + w_1 x_1 + w_2 x_2$$

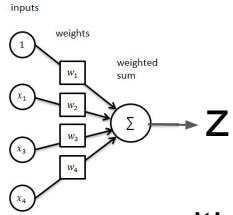
$$g(z) = \begin{cases} 0 \; if \;\; z < threshold \\ 1 \; if \;\; z \geq threshold \end{cases}$$



Example: Perceptron

- Goal: Get perceptron to predict classes from data that we can encode as 1 or 0 (e.g. pass/fail from continuous data of hours studied and slept)
- If we had a **regression problem**, we would have continuous labels (not just 1 and 0), and would build a model etc:

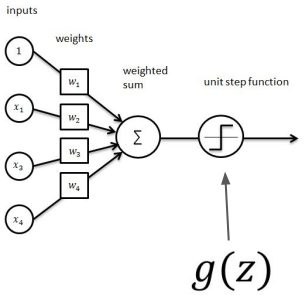
Studied	Slept	Passed
4.85	9.63	1
8.62	3.23	0
5.43	8.23	1
9.21	6.34	0



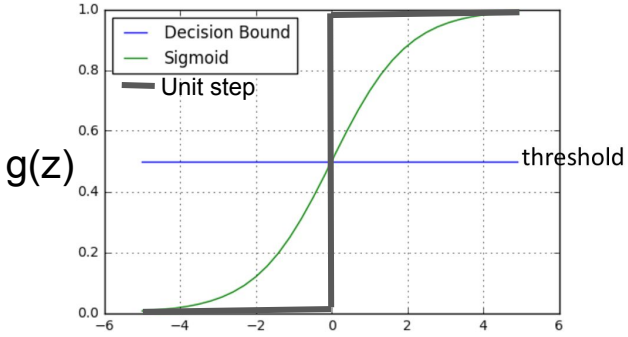
$$z = \mathbf{w}^T\mathbf{x} = w_0 1 + w_1 x_1 + w_2 x_2 + w_3 x + w_4 x_4$$

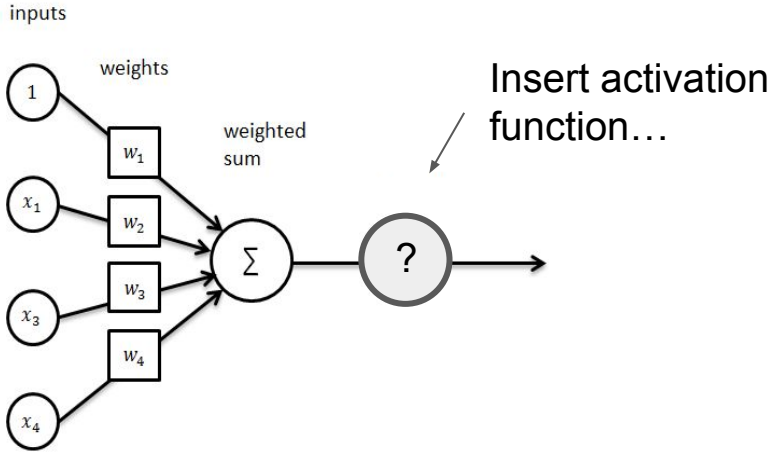
- However, with **classes** as labels we use a **threshold** encoded as an **activation function** g(z) in the perceptron, that takes z (providing continuous output) and returns a class:

$$g(z) = \begin{cases} 0 \; if \;\; z < threshold \\ 1 \; if \;\; z \geq threshold \end{cases}$$



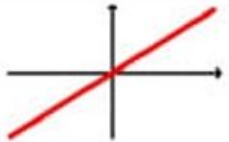
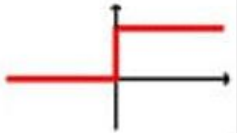
Changes in the activation function





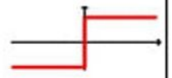
- The ‘activation function’ g(z) is what translates the z into the classes.
- We can change the function used in the activation function

Different activation functions we've seen…

Activation Function	Equation	Example	1D Graph
Linear	$\phi(z) = z$	Adaline, linear regression	
Unit Step (Heaviside Function)	$\phi(z) = \begin{cases} 0 & z < 0 \\ 0.5 & z = 0 \\ 1 & z > 0 \end{cases}$	Perceptron variant	
Sign (signum)	$\phi(z) = \begin{cases} -1 & z < 0 \\ 0 & z = 0 \\ 1 & z > 0 \end{cases}$	Perceptron variant	
Logistic (sigmoid)	$\phi(z) = \frac{1}{1 + e^{-z}}$	Logistic regression, Multilayer NN	

Activation functions continued.

- **Non-linearity in Neural Networks is introduced using Activation Functions**
- Allows learning complex patterns in data
- Determine the output of a NN
- Determine whether a neuron should “fire”
- Non-Linear activation functions help the model to generalize to a variety of data
- **A NN without an activation function is a linear regressor**

Activation Function	Equation	Example	1D Graph
Linear	$\phi(z) = z$	Adaline, linear regression	
Unit Step (Heaviside Function)	$\phi(z) = \begin{cases} 0 & z < 0 \\ 0.5 & z = 0 \\ 1 & z > 0 \end{cases}$	Perceptron variant	
Sign (signum)	$\phi(z) = \begin{cases} -1 & z < 0 \\ 0 & z = 0 \\ 1 & z > 0 \end{cases}$	Perceptron variant	
Piece-wise Linear	$\phi(z) = \begin{cases} 0 & z \le -½ \\ z + ½ & -½ \le z \le ½ \\ 1 & z \ge ½ \end{cases}$	Support vector machine	
Logistic (sigmoid)	$\phi(z) = \frac{1}{1 + e^{-z}}$	Logistic regression, Multilayer NN	
Hyperbolic Tangent (tanh)	$\phi(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$	Multilayer NN, RNNs	
ReLU	$\phi(z) = \begin{cases} 0 & z < 0 \\ z & z > 0 \end{cases}$	Multilayer NN, CNNs	

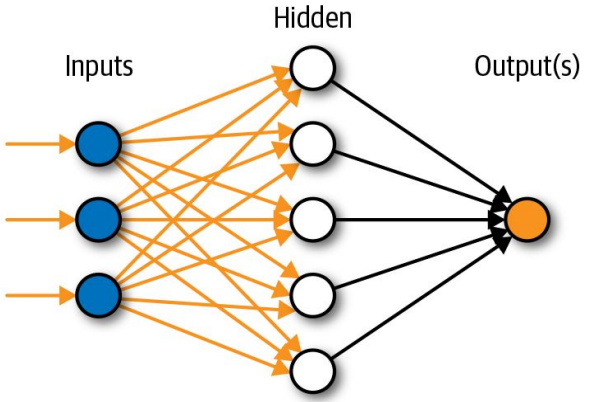
More on activation functions…

Formula	Output Range	Advantages	Disadvantages
Sigmoid $f(x) = 1 / (1 + e^{-x})$	0 to 1	• output a probability • Simple to compute	can cause vanishing gradient problem which can slow down training in deep networks (DNN)
ReLU $f(x) = \max(0, x)$	0 to $+\infty$	• Simple to compute • Non-saturating	can cause dying ReLU problem (neurons that output 0 for any input value < 0)
Leaky ReLU $f(x) = \max(\alpha x, x)$	$-\infty$ to $+\infty$	• Solves the dying ReLU problem • Non-saturating	
Tanh $f(x) = (e^{x} - e^{-x}) / (e^{x} + e^{-x})$	-1 to 1	• Useful when you need to output a value between -1 and 1 • mainly used for binary classification	can cause vanishing gradient problem which can slow down training in DNNs
Softmax $f(x_i) = \exp(x_i) / \Sigma(\exp(x_i))$	0 to 1	Useful to get output probabilities for multiple classes.	Can suffer from numerical instability.
Softplus $f(x) = \log(1 + e^{x})$	0 to $+\infty$	• Similar to ReLU, but smoother and differentiable everywhere, which can make it easier to optimize. • non-saturating • simple to compute.	Can cause vanishing gradient problem for large negative input values, which can slow down training in deep networks.
Swish $f(x) = x/(1 + e^{-\text{beta} * x})$	$-\infty$ to $+\infty$	• Can improve performance compared to ReLU and sigmoid. • Simple to compute.	Requires tuning the beta parameter.

Note: More details than you need in L6. We will return to activation functions when the context arises!

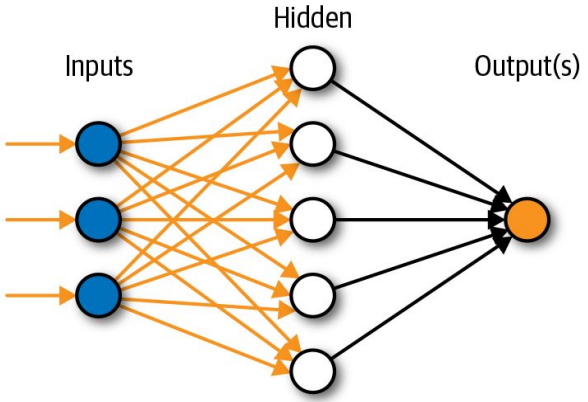
Neural Network: Layer context

- We will cover the neural network concepts:
  - layers, feedforward and ‘fully connected networks’
- Types of layers: input, hidden and output layer
- Feed-Forward NN (FFNN): composed of two or more layers (mostly 3), with activation functions that are usually step or logistic function
- Multi-Layer-Perceptron (MLP): has three or more layers
- Perceptron Learning is a NN without a hidden later, (i.e., two layers that are input and output)
  - Good for emulating the functionality of logical AND and OR, but not good for XOR problem (Boolean functions)



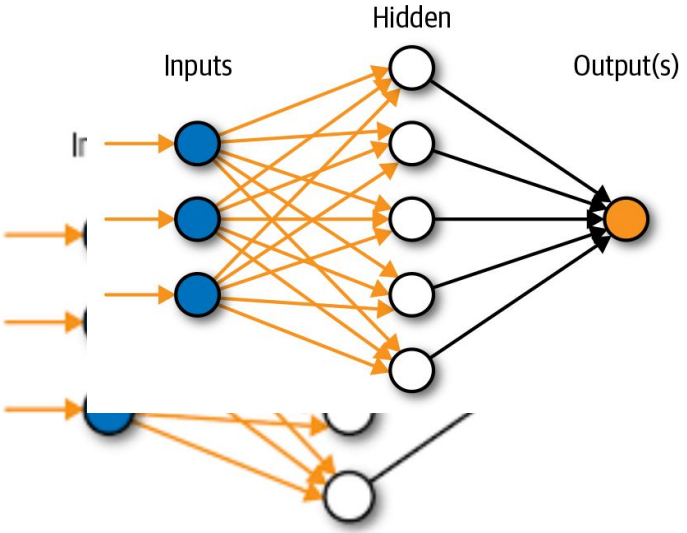
Neural Network: Layer context

- For most tasks 1-2 hidden layers are sufficient
- Too many hidden layers increases the complexity of the model and likely the training time, especially when the “error” is propagated backwards (backpropagation)
- With increasing numbers of hidden layers we have a “Deep” Neural Network (DNN)



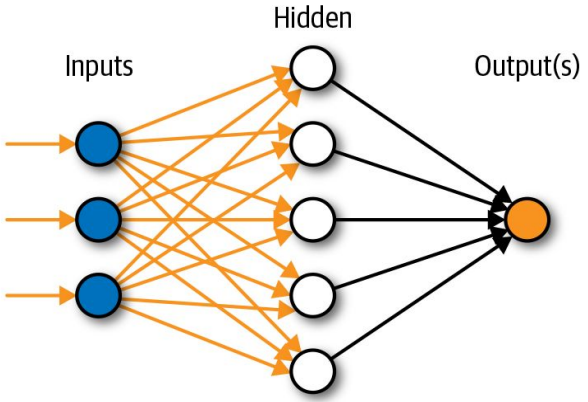
Feed-forward Neural Network (FFNN) : ‘Neurons’ or ‘Nodes’

- Number of nodes in the input layer depends on the number of dataset attributes
- The number of nodes in the output layer may be more than 1 depending on the classification task. How many nodes needed if the class variable has three labels?
- The number of nodes in the hidden layer depends on the complexity of the pattern. An overly large number of nodes can cause overfitting.
  - In case of overfitting, reduce the number of nodes in the hidden layer.
  - In case of low accuracy, increase the number of nodes in the hidden layer.
  - Determined by Trial-and-error

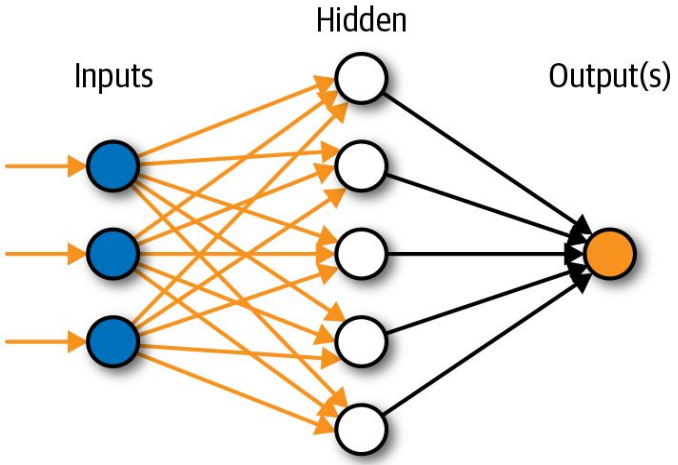


Classification Feed-forward Neural Network (FFNN) : ‘Neurons’ or ‘Nodes’

- Number of nodes in the input layer is given by the number of dataset attributes
- The number of nodes in the output layer may be more than 1 depending on the classification task
- The number of nodes in the hidden layer depends on the classification task (read: the data)
  - An overly large number of nodes can cause overfitting
  - If overfitting: reduce the number of nodes in the hidden layer
  - If low accuracy: increase the number of nodes in
  - the hidden layer
  - Determined by Trial-and-error



```python
import keras
from keras.models import Sequential
from keras.layers import Dense

# define the number of input nodes, hidden nodes, and output
nodes
input_dim = 5
hidden_dim = 10
output_dim = 1

# create a sequential model
model = Sequential()

# add the input layer with ReLU activation function
model.add(Dense(hidden_dim, input_dim=input_dim,
activation='relu'))

# add a hidden layer with ReLU activation function
model.add(Dense(hidden_dim, activation='relu'))

# add the output layer with sigmoid activation function
model.add(Dense(output_dim, activation='sigmoid'))

# compile the model with binary cross-entropy loss function
and Adam optimizer
model.compile(loss='binary_crossentropy', optimizer='adam')

# train the model on the training data
model.fit(X_train, y_train, epochs=100, batch_size=32,
validation_data=(X_test, y_test))
```



- A ‘fully connected’ Feedforward Neural Network (FFNN):
  - Every node is connected to every node in the next layer
  - Weights are valued between 0 and 1

Neural Networks (NN): Pros and Cons

Pros:

- Robust and resilient to noise
- Can model non-linearity

Cons:

- Black-box nature (mostly)
- Prone to overfitting
- Computationally intensive
- Data Requirement
- Hyperparameter tuning
- Standardizing all input attributes

```
>>> from sklearn.preprocessing import MinMaxScaler
>>> data = [[-1, 2], [-0.5, 6], [0, 10], [1, 18]]
>>> scaler = MinMaxScaler()
>>> print(scaler.fit(data))
MinMaxScaler()
>>> print(scaler.data_max_)
[ 1. 18.]
>>> print(scaler.transform(data))
[[0.   0.  ]
 [0.25 0.25]
 [0.5  0.5 ]
 [1.   1.  ]]
>>> print(scaler.transform([[2, 2]]))
[[1.5 0. ]]
```

Feature engineering for NNs: Normalising

- **Feature engineering is another name for pre-processing**
- Recall:
  - **Standardizing** with **z-score** (standard normal distribution with a mean (μ) of 0 and a standard deviation (σ) of 1), and
  - **Normalising aka min-max scaling** (X_{norm} rescales the feature to a fixed range, usually 0 to 1)
- NNs usually perform better with min-max scaling
  - Normalization is useful if know the distribution is not Gaussian or when you need to bound values
  - Note: normalisation is **sensitive to outliers** since the minimum and maximum values are used for scaling

$$z = \frac{(x - \mu)}{\sigma}$$

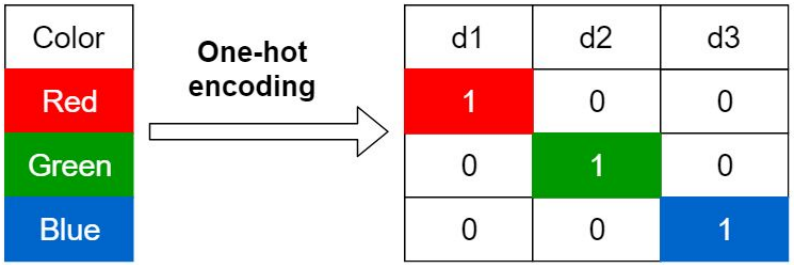
$$X_{\text{norm}} = \frac{(X - X_{\text{min}})}{(X_{\text{max}} - X_{\text{min}})}$$

Feature engineering for NNs: Categorical data

- Categorical variables allow a neural network to understand and represent discrete features of the data:
  - E.g.: Day of the week, color, country
- Learning the representations of categories is largely more generalisable with NNs
- Label Encoding assigns a unique numerical value to categorical data.
  - It assumes an ordered relationship between the categories, so it is not good for encoding nominal attributes
  - Example: Feedback variable taking labels of ‘bad’, ‘good’, and ‘excellent’ taking values of ‘0’,’1’, ‘2’
  - Not recommended for NN due to introducing bias; one-hot encoding is more preferred.
  - Drawback: Arbitrary Ordering can lead to incorrect interpretations

Feature engineering for NNs: Categorical data

- One-hot Encoding is a technique used to represent categorical data as numerical input (i.e., binary vectors) in machine learning.
  - Each label/category in a categorical variable (such as ‘color’) is represented as a binary vector with one element set to 1 and all other elements set to 0
- Example one-hot encoding:
  - Attribute ‘color’ can be ‘red’,’green’, and ‘blue’. The number of labels is 3. You need (k-1 = 2) ‘flag variables’
  - Using one-hot encoding, you can remove ‘color’ column from the dataset and add three columns of ‘red’ , ‘green’ and ‘blue’
- Drawback: it can lead to high-dimensional sparse data, where most of the elements in the encoded vector are zero



One-Hot vs. label encoder

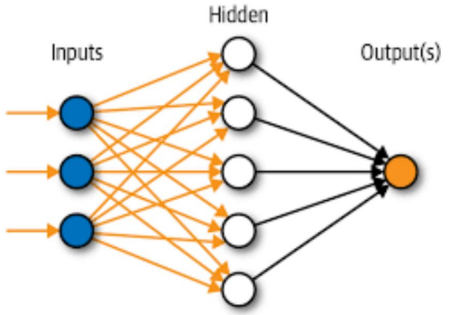
```
from sklearn.preprocessing import
OneHotEncoder
import numpy as np

# create a sample categorical data
data = np.array(['red', 'green', 'blue',
'blue', 'red']).reshape(-1, 1)

# create an instance of the encoder
encoder = OneHotEncoder()

# fit the encoder to the data and transform
the data
data_encoded = encoder.fit_transform(data)

# convert the encoded data to a numpy array
data_encoded = data_encoded.toarray()

# print the encoded data
print(data_encoded)
```

```
[[0. 0. 1.]
[0. 1. 0.]
[1. 0. 0.]
[1. 0. 0.]
[0. 0. 1.]]
```

```
from sklearn.preprocessing import
LabelEncoder
import numpy as np

# create a sample categorical data
data = np.array(['red', 'green', 'blue',
'blue', 'red']).reshape(-1, 1)

# create an instance of the encoder
encoder = LabelEncoder()

# fit the encoder to the data and transform
the data
data_encoded =
encoder.fit_transform(data.ravel())

# print the encoded data
print(data_encoded)
```

```
[2 1 0 0 2]
```

One-output Node Application

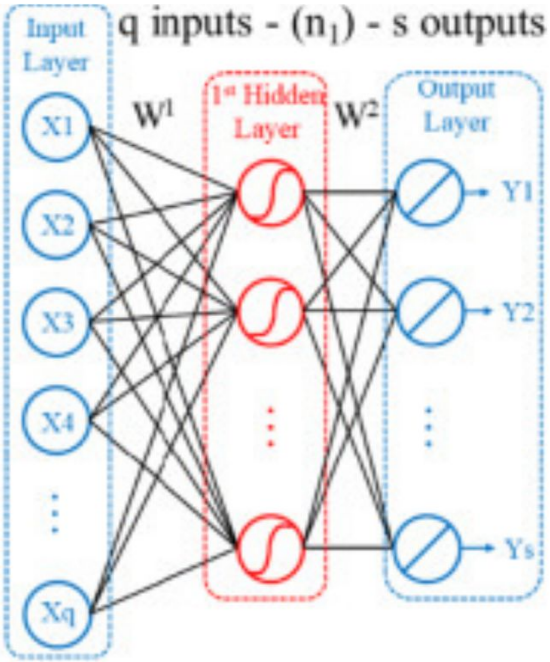


- Good for binary classification (0/1 or win/lose)
- One output node is also good when the output classes are ordered
  - In this case, works for multi-class classification
  - Example: 1, 2, or 3rd place
  - The ‘order’ is used as a continuous or discrete numeric value

$$
\begin{aligned}
&if\ 0 \le output < threshold1 && : classify\ 1st\ place \\
&if\ threshold1 \le output < threshold2 && : classify\ 2nd\ place \\
&if\ threshold2 \le output && : classify\ 3rd\ place
\end{aligned}
$$

1-of-n output Encoding

- There is more than one output node in the output layer (Y)
- Output classes are not ordered (i.e., nominal) e.g., gender: {male, female, unknown}
- Each output node corresponds to one class label, quantified with a probability
- Benefit: it provides probabilities used as a measure of **confidence** in the classification



Feed-Forward Neural Network: Combination Function

- The ‘feedforward’ combination function in a neural network is the operation used to compute the input to a neuron in a given layer from the outputs of the neurons in the previous layer
- The equation describes how data flows through the network during the forward pass
- First: Linear Combination of Inputs: $z = w_1x_1 + w_2x_2 + \ldots + w_nx_n + b$
  - Here, z values are also known as the neuron's ‘pre-activation’ values, and referred to as a ‘net’, sum, or Σ
- Second: Introduce the activation function to z
  - Pass the z, or ‘pre-activation’ values to the activation function (e.g. sigmoid, unit step)
- Third: Propagate to the next layer

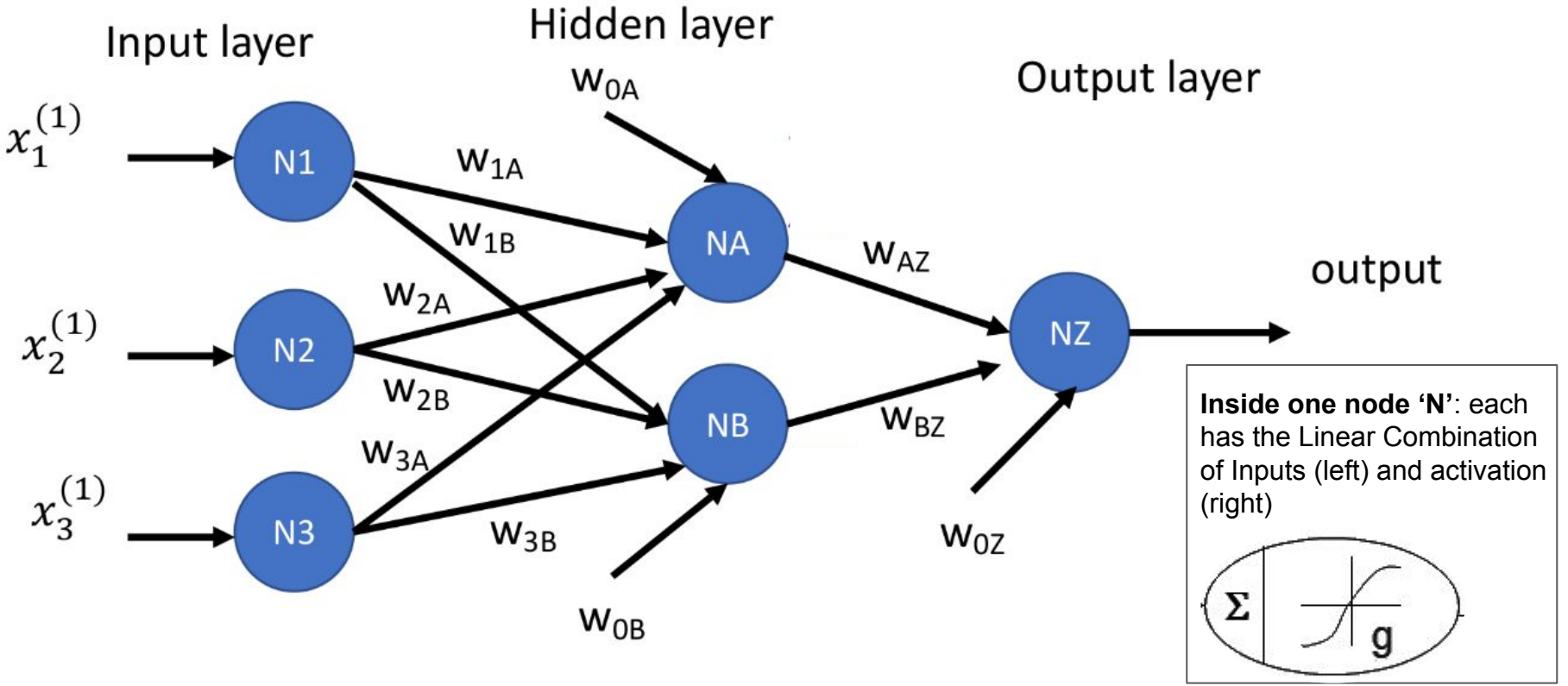
Feed-Forward Neural Network: Combination Function

Nx4 input matrix
N: number of samples
4: number of attributes including x_0
$x_0 \; is \; 1 \; by \; convention$

$$\begin{bmatrix} x_0^{(1)} & x_1^{(1)} & x_2^{(1)} & x_3^{(1)} \\ \vdots & & \ddots & \vdots \\ x_0^{(n)} & & \cdots & x_3^{(n)} \end{bmatrix} = \begin{bmatrix} 1 & x_1^{(1)} & x_2^{(1)} & x_3^{(1)} \\ \vdots & & \ddots & \vdots \\ 1 & & \cdots & x_3^{(n)} \end{bmatrix}$$

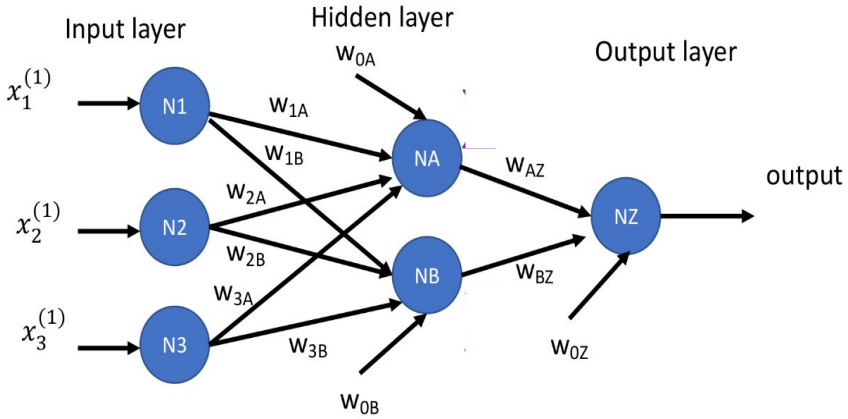
$$y^{(i)} = \sum_{j=0}^{M} w_j x_j^{(i)} = \omega_0 + \omega_1 x_1^{(i)} + \omega_2 x_2^{(i)} + \omega_3 x_3^{(i)}$$

Feed-Forward Neural Network: Combination Function



Feed-Forward Neural Network: Combination Function

$$\begin{bmatrix} x_0^{(1)} & x_1^{(1)} & x_2^{(1)} & x_3^{(1)} \\ \vdots & & \ddots & \vdots \\ x_0^{(n)} & & \cdots & x_3^{(n)} \end{bmatrix} = \begin{bmatrix} 1 & x_1^{(1)} & x_2^{(1)} & x_3^{(1)} \\ \vdots & & \ddots & \vdots \\ 1 & & \cdots & x_3^{(n)} \end{bmatrix}$$



Initiate weights randomly

w_{0A} =0.5	w_{0B} =0.7	w_{0Z}=0.5
w_{1A} = 0.6	w_{1B} = 0.9	w_{AZ}=0.9
w_{2A}=0.8	w_{2B}=0.8	w_{BZ}=0.9
w_{3A}=0.6	w_{3B}=0.4	

w_{ij}: the weight associated with the i^{th} input to node j

x_{ij}: i^{th} input to node j

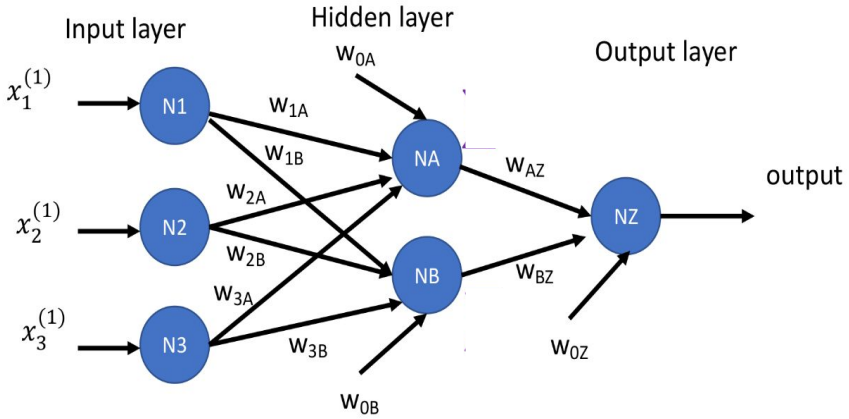
Feed-Forward Neural Network: Combination Function

$$\begin{bmatrix} x_0^{(1)} & x_1^{(1)} & x_2^{(1)} & x_3^{(1)} \\ \vdots & & \ddots & \vdots \\ x_0^{(n)} & & \cdots & x_3^{(n)} \end{bmatrix} = \begin{bmatrix} 1 & x_1^{(1)} & x_2^{(1)} & x_3^{(1)} \\ \vdots & & \ddots & \vdots \\ 1 & & \cdots & x_3^{(n)} \end{bmatrix}$$

Initiate weights randomly

w_{0A} =0.5	w_{0B} =0.7	w_{0Z}=0.5
w_{1A} = 0.6	w_{1B} = 0.9	w_{AZ}=0.9
w_{2A}=0.8	w_{2B}=0.8	w_{BZ}=0.9
w_{3A}=0.6	w_{3B}=0.4	

w_{ij}: the weight associated with the i^{th} input to node j
x_{ij}: i^{th} input to node j



Calculate the scalar value passed to a node j through Linear Combination of Inputs:

$$net^{(i)}{}_j = \sum_k w_{kj} x_{kj}^{(i)}$$

For node NA:

$$net^{(1)}{}_A = \omega_{0j} + \omega_{1j} x_{1j}^{(1)} + \omega_{2j} x_{2j}^{(1)} + \omega_{3j} x_{3j}^{(1)}$$

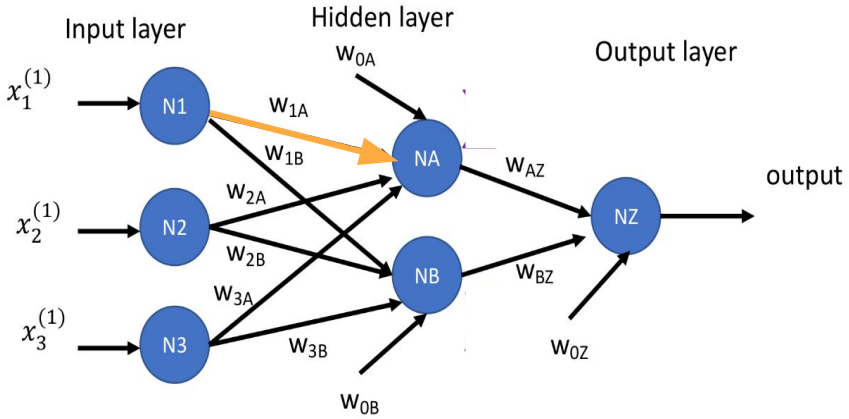
Feed-Forward Neural Network: Combination Function

$$\begin{bmatrix} x_0^{(1)} & x_1^{(1)} & x_2^{(1)} & x_3^{(1)} \\ \vdots & & \ddots & \vdots \\ x_0^{(n)} & & \cdots & x_3^{(n)} \end{bmatrix} = \begin{bmatrix} 1 & x_1^{(1)} & x_2^{(1)} & x_3^{(1)} \\ \vdots & & \ddots & \vdots \\ 1 & & \cdots & x_3^{(n)} \end{bmatrix}$$

Initiate weights randomly

w_{0A} =0.5	w_{0B} =0.7	w_{0Z}=0.5
w_{1A} = 0.6	w_{1B} = 0.9	w_{AZ}=0.9
w_{2A}=0.8	w_{2B}=0.8	w_{BZ}=0.9
w_{3A}=0.6	w_{3B}=0.4	

w_{ij}: the weight associated with the i^{th} input to node j
x_{ij}: i^{th} input to node j



Calculate the scalar value passed to a node j through Linear Combination of Inputs:

$$net^{(i)}{}_j = \sum_k w_{kj} x_{kj}^{(i)}$$

For node NA:

$$net^{(1)}{}_A = \omega_{0j} + \omega_{1j} x_{1j}^{(1)} + \omega_{2j} x_{2j}^{(1)} + \omega_{3j} x_{3j}^{(1)}$$

Feed-Forward Neural Network: Combination Function

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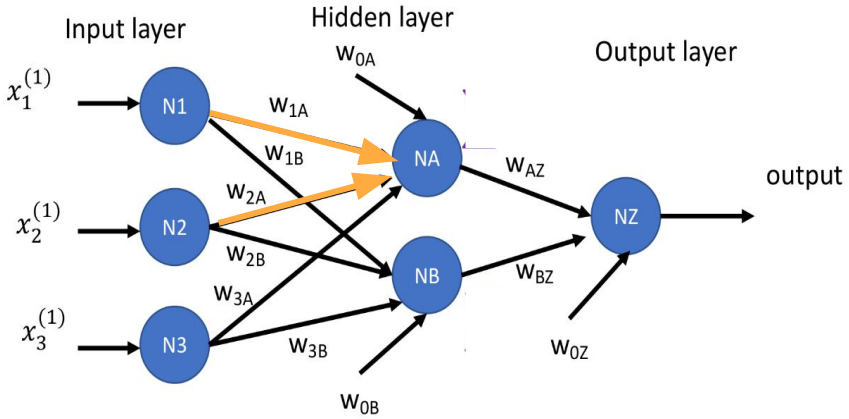

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Feed-Forward Neural Network: Combination Function

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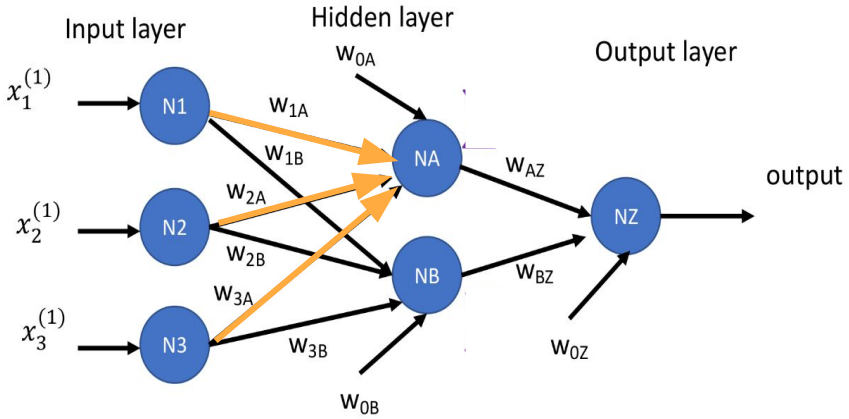

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Feed-Forward Neural Network: Combination Function

$$\begin{bmatrix} x_0^{(1)} & x_1^{(1)} & x_2^{(1)} & x_3^{(1)} \\ \vdots & & \ddots & \vdots \\ x_0^{(n)} & & \cdots & x_3^{(n)} \end{bmatrix} = \begin{bmatrix} 1 & x_1^{(1)} & x_2^{(1)} & x_3^{(1)} \\ \vdots & & \ddots & \vdots \\ 1 & & \cdots & x_3^{(n)} \end{bmatrix}$$

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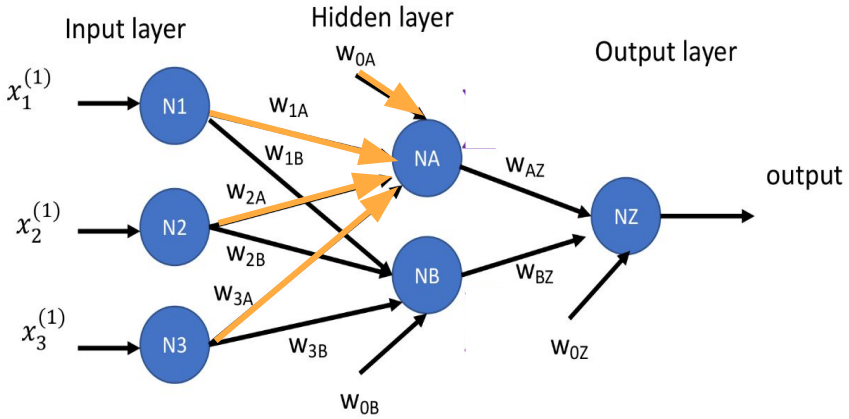

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Feed-Forward Neural Network: Combination Function

$$\begin{bmatrix} x_0^{(1)} & x_1^{(1)} & x_2^{(1)} & x_3^{(1)} \\ \vdots & \ddots & & \vdots \\ x_0^{(n)} & \cdots & & x_3^{(n)} \end{bmatrix} = \begin{bmatrix} 1 & x_1^{(1)} & x_2^{(1)} & x_3^{(1)} \\ \vdots & \ddots & & \vdots \\ 1 & \cdots & & x_3^{(n)} \end{bmatrix}$$

Initiate weights randomly

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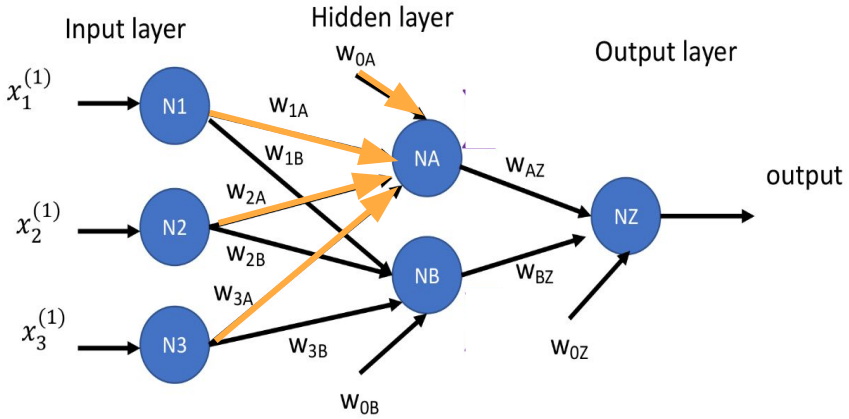


Calculate the scalar value passed to a node j through Linear Combination of Inputs:

$$net^{(i)}{}_j = \sum_k w_{kj} x_{kj}^{(i)}$$

For node NA:

$$net^{(1)}{}_A = \omega_{0j} + \omega_{1j} x_{1j}^{(1)} + \omega_{2j} x_{2j}^{(1)} + \omega_{3j} x_{3j}^{(1)} = 1.32$$

Feed-Forward Neural Network: Combination Function

Our scalar value passed to a node NA through Linear Combination of Inputs:

$$net^{(1)}{}_A = \omega_{0j} + \omega_{1j}x_{1j}^{(1)} + \omega_{2j}x_{2j}^{(1)} + \omega_{3j}x_{3j}^{(1)} = 1.32$$

The input (1.32) is now given to the activation function g e.g. a sigmoid such that in general:

$$y = \frac{1}{1+e^{-z}} = g(net_j)$$

For NA, the result now becomes:

$$g(net_A) = \frac{1}{1+e^{-1.38}} = 0.7892$$

Similarly for NB, going through the Linear Combination of Inputs for node NB:

$$g(net_B) = \frac{1}{1+e^{-net_B}} = 0.8176$$

We then for the *next* layer would repeat the process until we reach the output layer…

Feed-Forward Neural Network: Combination Function

w_{0A} =0.5	w_{0B} =0.7	w_{0Z}=0.5
w_{1A} = 0.6	w_{1B} = 0.9	w_{AZ}=0.9
w_{2A}=0.8	w_{2B}=0.8	w_{BZ}=0.9
w_{3A}=0.6	w_{3B}=0.4	

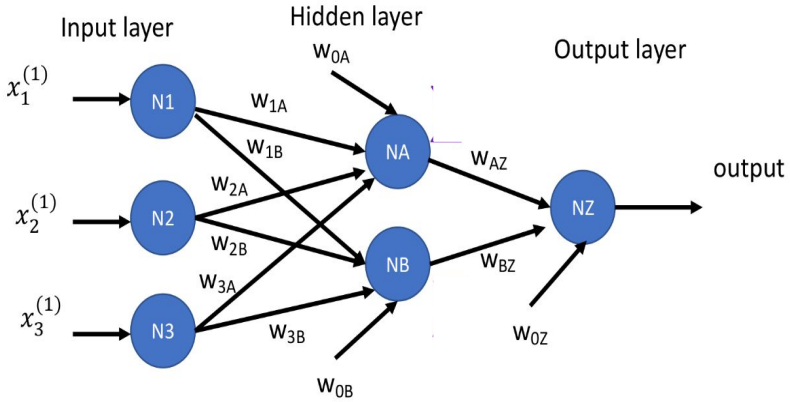
From NA and NB we have 0.7892 and 0.8176 (effective our x_{AZ} and x_{BZ}) and weights w_{0Z} , w_{AZ} and w_{BZ} are in the table

We calculate the Linear Combination of Inputs in NZ as:

$$net^{(1)}{}_Z = \sum_k w_{kZ} x_{kZ}^{(1)} = \omega_{0Z} + \omega_{AZ} x_{AZ}^{(1)} + \omega_{BZ} x_{BZ}^{(1)} = 0.5 + 0.9(0.7892) + 0.9(0.8176) = 1.9461$$

This again is passed to the activation function g:

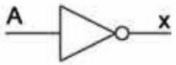
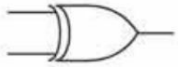
$$g(net_Z) = \frac{1}{1 + e^{-1.9461}} = 0.8750$$



Output from the NN for pass 1 through the network, and it is the predicted value for the first observation in the dataset D.

Emulating Boolean Functions with a NN

Logical Gates

Name	NOT	AND	NAND	OR	NOR	XOR	XNOR
Alg. Expr.	$\overline{A}$	AB	$\overline{AB}$	$A+B$	$\overline{A+B}$	$A \oplus B$	$\overline{A \oplus B}$
Symbol							
Truth Table	A X 0 1 1 0	B A X 0 0 0 0 1 0 1 0 0 1 1 1	B A X 0 0 1 0 1 1 1 0 1 1 1 0	B A X 0 0 0 0 1 1 1 0 1 1 1 1	B A X 0 0 1 0 1 0 1 0 0 1 1 0	B A X 0 0 0 0 1 1 1 0 1 1 1 0	B A X 0 0 1 0 1 0 1 0 0 1 1 1

Source: https://medium.com/autonomous-agents/how-to-teach-logic-to-your-neuralnetworks-116215c71a49